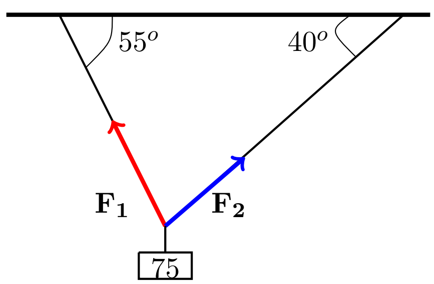
Applications of Vectors

1. Velocity and Speed. The magnitude of a velocity vector is called speed.

   Example: A quarterback throws a football with an angle of elevation 60^o and speed 50 ft/s. Find the horizontal and vertical components of the velocity vector.

2. Force. A force is represented by a vector because it has both magnitude and a direction. If several forces are acting on an object, the **resultant force** experienced by the object is the vector sum of these forces.

   Example: A 75-N weight is suspended by two wires, as shown below. Find the forces $\mathbf{F_1}$ and $\mathbf{F_2}$ acting in both wires.



DNF

Note: The resultant $\mathbf{F_1} + \mathbf{F_2}$ counterbalances the weight $\mathbf{w} = \langle 0, -75\rangle$.

We have $\mathbf{F_1} + \mathbf{F_2} = -\mathbf{w}$

$$\langle -|F_1|\cos(55^\circ), |F_1|\sin(55^\circ)\rangle + \langle |F_2|\cos(40^\circ), |F_2|\sin(40^\circ)\rangle = -\langle 0, -75\rangle.$$

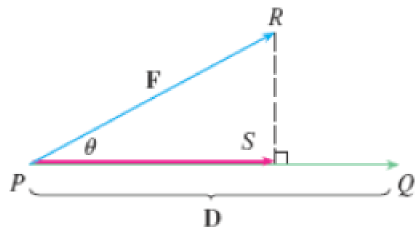
$$\begin{cases} -|F_1|\cos(55^\circ) + |F_2|\cos(40^\circ) = 0 \\ |F_1|\sin(55^\circ) + |F_1|\sin(40^\circ) = 75 \end{cases} \rightarrow \quad |F_2| = \frac{|F_1|\cos(55^\circ)}{\cos(40^\circ)}$$

then $$|F_1|\sin(55^\circ) + \frac{|F_1|\cos(55^\circ)}{\cos(40^\circ)}\sin(40^\circ) = 75.$$

$$\Rightarrow \quad |F_1| = \frac{75}{\sin(55^\circ) + \frac{\cos(55^\circ)\sin(40^\circ)}{\cos(40^\circ)}} = ?$$

DNF

3. Work. Suppose the force is a vector $\mathbf{F} = \overrightarrow{PR}$ pointing in some direction, as below. If the force moves the object from P to Q, then the *displacement vector* is $\mathbf{D} = \overrightarrow{PQ}$.



The **work** done by this force is defined to be the product of the component of the force along $\mathbf{D}$ and the distance moved:

$$\mathbf{W} = |\mathbf{F}||\mathbf{D}|\cos(\theta) = \mathbf{F}\cdot\mathbf{D}$$

Example: A sled is pulled along a level path through snow by a rope. A 25-lb force acting at an angle of 45^o above the horizontal moves the sled 60 ft. Find the work done by the force.

$|F| = 25$ lb

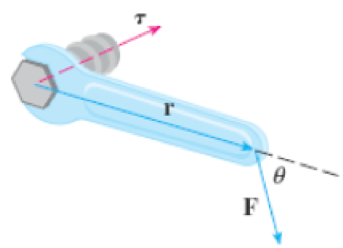
$\theta = 45^\circ$

$|D| = 60$ ft

$W = |F||D|\cos\theta \Rightarrow (25)(60)\frac{\sqrt{2}}{2} = \underline{750\sqrt{2} \text{ ft-lb}}$

4. Torque. The **torque** τ (relative to the origin) is defined to be the cross product of the position and force vectors, and measures the tendency of the body to rotate about the origin.

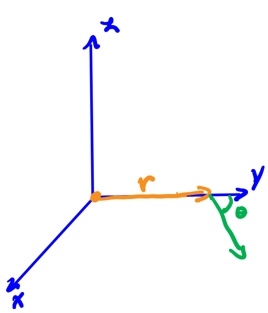
$\tau = \mathbf{r} \times \mathbf{F}$



The magnitude of the torque vector is

$|\tau| = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}||\mathbf{F}|\sin(\theta)$

Example: A wrench 30cm long lies along the positive y-axis and grips a bolt at the origin. A force is applied in the direction $\langle 0, 3, -4\rangle$ at the end of the wrench. Find the magnitude of the force needed to supply 100N·m of torque to the bolt.



$||r|| = 30\text{cm} = 0.3\text{m}$

$F = \lambda\langle 0, 3, -4\rangle$ Goal: $|F| = \lambda(5) = ?$

$||\tau|| = 100$

Note that $|\tau| = |r||F|\sin\theta$

$100 = (0.3)(5\lambda)\frac{4}{5} \Rightarrow 5\lambda = \frac{5(100)}{4(0.3)}$

So, magnitude of the force is $|F| = 5\lambda = \frac{5(100)}{4(0.3)}$